The improved 10th order QED expresion for a_μ: new results and related estimates

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New estimates of the 10th order QED corrections to the muon anomalous magnetic moment are presented. The estimates include the information on definite improved 10th order QED contributions to a_μ, calculated by Kinoshita and Nio. The final estimates are in good agreement with the ones, given recently by Kinoshita.

1. INTRODUCTION

In the last years both theoretical and experimental results for the anomalous magnetic moment a_μ attracted special interest (for the most recent review see Ref. [1]).

Careful analysis of the values of different theoretical corrections to a_μ stimulated the new fresh glance on the pure QED expression for this classical quantity. The work was started after definite bugs in the previous calculations of eighth-order light-by-light-type diagrams [2] were detected and corrected [3]. The evaluations of all mass-dependent α^4 QED contributions to a_μ were completed in Ref. [4] and their numerical values have been greatly improved with respect to previous results of Ref. [2].

Moreover, the crude estimate of the α^5 QED correction to a_μ, which is based on the calculations of the dominant contributions to the sets of 10th order light-by-light-type diagrams (see Ref. [2] and Ref. [5]) and the renormalization group inspired studies of Refs. [2,6] was also improved [7]. In view of this it is worthwhile to reconsider the 10th order scheme-invariant estimates of Ref. [8], which were in qualitative agreement with the estimate from Ref.[6].

*Supported by RFBR Grants N 03-02-17177, 05-01-00992

2. FEYNMAN CHALLENGE

The problem of estimates of high order perturbative corrections to physical quantities was first formulated by R. Feynman. In the talk at the 1961 Solvey Conference he mentioned : “As a special chellenge, is there any method of computing the anomalous moment of the electron which, on the first rough approximation, gives a fair approximation to the α-term and a crude one to α^2, and when improved, increases the accuracy of the α^2 term, yielding a rough estimate to α^3 and beyond ?” [9]. This question reveales the useful features of theoretical estimates. At the first stage they may give the impression on the sign-structure of perturbative series, at the second stage are stimulating studies of the effects, not included in these estimates, which when calculated and improved at the third stage are giving the final result for the whole correction.

2.1. Anomalous magnetic moment of muon: 8th order QED results

The general QED expression for a_μ is :

$$\begin{aligned} a_\mu &= a_e + A_2(m_\mu/m_e) \\ &+ A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau) \end{aligned} \quad (1)$$

where

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots \quad (2)$$

and $i = 1, 2, 3$. The first three corrections to a_e are known in the analytical form from the calcu-

lations of Refs. [10]- [12]. The updated value of the 8th order correction to a_e was presented in Ref. [7].

The dominant numerical values of the terms $A_2^{(4)}$ and $A_2^{(6)}$ are known and read [1,7] :

$$A_2^{(4)}(m_\mu/m_e) = 1.0942582887(104) , \quad (3)$$
$$A_2^{(6)}(m_\mu/m_e) = 22.86837936(22) . \quad (4)$$

Other terms in Eq. (2) are rather small and are of order 10^{-4}- 10^{-5} [1,7]. The re-evaluation of the 8th order contributions to a_μ gives the improved number [4], namely :

$$A_2^{(8)}(m_\mu/m_e) = 132.6823(72) \quad (5)$$

Notice, that the coefficients of $A_2(m_{\mu/m_e})$ are positive and their values are increasing. This happens due to the contributions of the powers of the relatively large renormalization-group (RG) controllable terms with $\ln(m_\mu/m_e) \approx 5.6$. Moreover, beginning from the 6th order the light-by-light-type diagrams with internal fermion loop are starting to manifest themselves [14]. Their typical contribution are proportional to $\pi^2 \ln(\mathrm{m}_\mu/\mathrm{m}_\mathrm{e})$–factors, which have non-RG origin and are dominating in the expressions for the corresponding coefficients of the 8th order correction. Thus, one may expect, that they will continue to dominate in higher orders also.

2.2. 10th order QED corrections to a_μ

The first estimate of the 10th-order correction to a_μ was given in Ref. [2] on the basis of rather preliminary numerical evaluation of the 10th-order diagrams with electron light-by-light subgraph and two one-loop electron vacuum polarization insertions into internal virtual photons, coupled to the muon line. This estimate reads [2]

$$\Delta_1(A_2^{(10)}) \approx 570(140) \quad . \quad (6)$$

However, there are at least two other sets of diagrams which were not taken into account in the estimate of Eq.(6) and may give sizable contribution. Among them is the light-by-light- type diagram, where one of three photons contains two-loop electron vacuum polarization insertion. Its contribution was estimated in Ref. [6] and reads

$$\Delta_2(A_2^{(10)}) \approx 176(35) \quad . \quad (7)$$

In the same work the contribution to $A_2^(10)$ of the diagram with electron loop, coupled to muon line by five photons, was estimates as [6] :

$$\Delta_3(A_2^{(10)}) \approx 185(85) \quad . \quad (8)$$

Eq.(8) includes theoretical and numerical information, gained from Refs. [5]. Summing up the estimates of Eq. (6) - Eq.(8) one can get [6]

$$\Delta_4(A^{(10)}) = \Delta_1 + \Delta_2 + \Delta_3 \quad \approx 930(170) \quad . \quad (9)$$

Another, more theoretical estimate, was made in Ref. [8]. It is based on application of the scheme-invariant methods, namely the principle of minimal sensitivity [15] or the effective charges method [16]. In the estimates of Ref. [8] the information on the values of lower-order contributions to a_μ (up to 8th order) and on the four-loop expression for the QED β-function in the on-shell scheme [18] were used. The developed approach, when applied separately to the sets of non-light-by-light terms and the sum of light-by-light-type contributions, gave the following numbers [8]

$$\Delta_1^{ECH}(A_2^{(10)}) \approx 50 \quad (10)$$
$$\Delta_2^{ECH}(A_2^{(10)}) \approx 521 \quad . \quad (11)$$

Note, that Eq.(11) contains the estimates for the sum of several 10th order contributions, including the ones, estimated separately within other approaches in Eq.(6) and Eq. (7). However, to obtain the final estimate within this scheme-invariant method it is also necessary to add the contribution of Eq. (8). Thus the estimate of the 10th order QED correction to a_μ, obtained in Ref. [8], was

$$\Delta_3^{ECH}(A^{(10)}) = \Delta_1^{ECH} + \Delta_2^{ECH} + \Delta_3 \approx 750 \, . \quad (12)$$

Within existing theoretical uncertainties the number of Eq. (12) do not contradict to the one of Eq. (9).

However, quite recently more detailed 10th order results, based on the calculations of Kinoshita and Nio [17], were announced [7]. These results are:

$$\Delta_1(A_2^{(10)}) = 629.1407(118) \quad (13)$$
$$\Delta_2(A_2^{(10)}) = 181.1285(51) \quad (14)$$
$$\Delta_3(A_2^{(10)}) = 86.69 \quad (15)$$

Kinoshita and Nio also calculated several other sets of 10th order diagrams diagrams, including the ones evaluated previously in Refs. [18]- [21]. The new estimate, which is based on the calculated part of 9080 diagrams, contributing to the the 10th order QED contribution, is [7]:

$$\Delta_{new}(A_2^{(10)}) = 677(40) \quad . \tag{16}$$

Note, that the calculations of the terms estimated in Eq. (8) are leading to the essential reduction of their contribution into the 10th order correction to a_μ (compare Eq. (15) with Eq. (8). Taking into account the effect of reduction of the contribution of Δ_3 into Eq. (12) we obtain a new estimate

$$\Delta_{new}^{ECH}(A_2^{(10)}) \approx 658 \tag{17}$$

which is in perfect agreement with the estimate of Eq.(16), based on explicit calculations of Ref. [17]. We believe, that this good agreement is not the accident and is demonstrating that both theoretical logic of scheme-invariant methods and the results of exact calculations are in good shape and are supporting each other. More detailed analysis of these results will be presented elsewhere.

As to phenomenological consequence, the agreement of the preliminary partial results of 10th order calculations to a_μ with the scheme-invariant result of Eq. (16) demonstrates, that the uncertainties of the 10th order QED contributions to a_μ are really small. However, there is the possibilities of decreasing current theoretical uncertainties to a_μ. It can be done as the result of taking into account in the calculations of the hadronic vacuum polarization contributions (for their evaluation see e.g. the reviews of Refs. [1],[22]) new data in the low energy region, which will be obtained soon at Novosibirsk e^+e^- collider, and to rely on possible reconstruction of DAPHNE (Frascati) machine with the aim to measure the region in e^+e^- -annihilation cross-section, complementary to the one, studied at Novosibisrk and Bejing colliders.

ACKNOWLEDGEMENTS This talk was prepared during the visit to ICTP (Trieste). I am grateful to the staff of this Center for providing excellent conditions for work. I also thank organizers of NuFact05 Workshop and V. Palladino in particular for invitation and hospitality in Frascati. I am grateful to V. Starshenko for the interest in the current status of different studies, related to a_μ and to T. Kinoshita for useful correspondence.

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